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CORRECTION SCHEMES FOR THE NORMAL OCTUPOLE AND DECAPOLE ERRORS IN THE LHC DIPOLES

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Abstract

Different correction schemes for curing the effect of the normal octupole and decapole multi-pole errors of the LHC main dipoles, at injection, are investigated. Frequency and diffusion maps are constructed and compared for two working points and for different values of the momentum deviation. The excitation of individual resonant driving terms is estimated through high order normal form construction techniques.

1 CORRECTION SCHEMES

The main limitation for the stability of the LHC [1] beam at the injection energy of 450 GeV is the magnet imperfections in the 1104 super-conducting arc (main) dipoles which introduce non-linear fields expressed in the usual complex multi-pole expansion:

$$B_y + i\,B_x = B_1 \sum_{n=1}^{\infty} (b_n + i a_n) \left(\frac{x + i\,y}{R_r}\right)^{n-1} , \quad (1)$$

where B_x, B_y are the horizontal and vertical components of the magnetic field, B_1 is the magnetic dipole field in the vertical y direction, $R_r = 17$mm the reference radius and the normal (or erect) b_n and skew a_n multipole coefficients. The multi-polar components responsible for the perturbations from the ideal dipole field are due to the persistent currents in the filaments of the super-conductor, the design geometry and the fabrication tolerances. Taking into account all the previous effects, error tables are estimated and used for beam dynamics analysis. The most important errors aloud by the dipole symmetry are the normal sextupole b_3 and decapole b_5. The normal octupole b_4 has a non-negligible effect due to the geometric imperfections induced by the two-in-one form of the LHC dipoles. These multi-pole coefficients corresponding to the 9901 error table are displayed in Table 1. The dominant normal sextupole error is planed to be corrected with magnetic coils (spool pieces) placed at the entrance of each dipoles and powered in series in each arc, in order to cancel locally the effect. The same strategy can be also followed for the octupole and decapole, where the correction is done with a spool piece having both components.

Previous studies for the erect octupole [2] and decapole [3] correction have shown that the dynamic aperture of the LHC at injection could be preserved even if half of the correctors are used. In order to validate these results for LHC optics version 6, the following correction schemes have been considered [4]:

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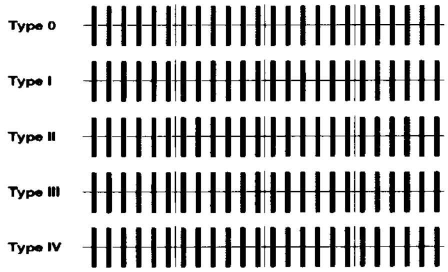

Figure 1: Representation of five different correction schemes in four cells of the LHC. The dipoles with and without octupole/decapole corrector are shown in blue and red, respectively.

- Type O: Correctors at all dipoles in all arcs; this is the reference case.
- Type I: Correctors in every second dipole; a scheme proposed for SSC and it is an interesting option for the LHC as it minimizes the electrical noise.
- Type II: Correctors in every second cell; it is a proposed layout worked out to balance the impedance on the bus-bars.
- Type III: Correctors in every second dipole with a swap of the two types of dipoles every second cell; this arrangement should be also better than the reference case one with respect to electrical noise.
- Type IV: Same as Type III but the two types of dipoles are inverted.

A graphical representation of the correction schemes can be found in Fig. 1, where we represent the dipoles with and without octupole/decapole correctors in four cells of the LHC arc (the quadrupoles are omitted).

In this paper we present the studies undertaken for the dynamic analysis of the different correction schemes. Frequency and diffusion maps [5] were constructed for comparing their impact to non-linear dynamics. In order to check the efficiency of each scheme on resonance compensation, the resonance driving terms are computed, through high-order perturbation theory methods [6] and numerical post-processors [7].

Table 1: Erect sextupole, octupole and decapole field errors in the LHC dipoles (error table 9901), for a reference radius of $R_r = 17$mm in units of 10^{-4} of the main field.

Error	Systematic	Random
b_3	-8.32	1.47
b_4	0.57	0.51
b_5	1.32	0.43

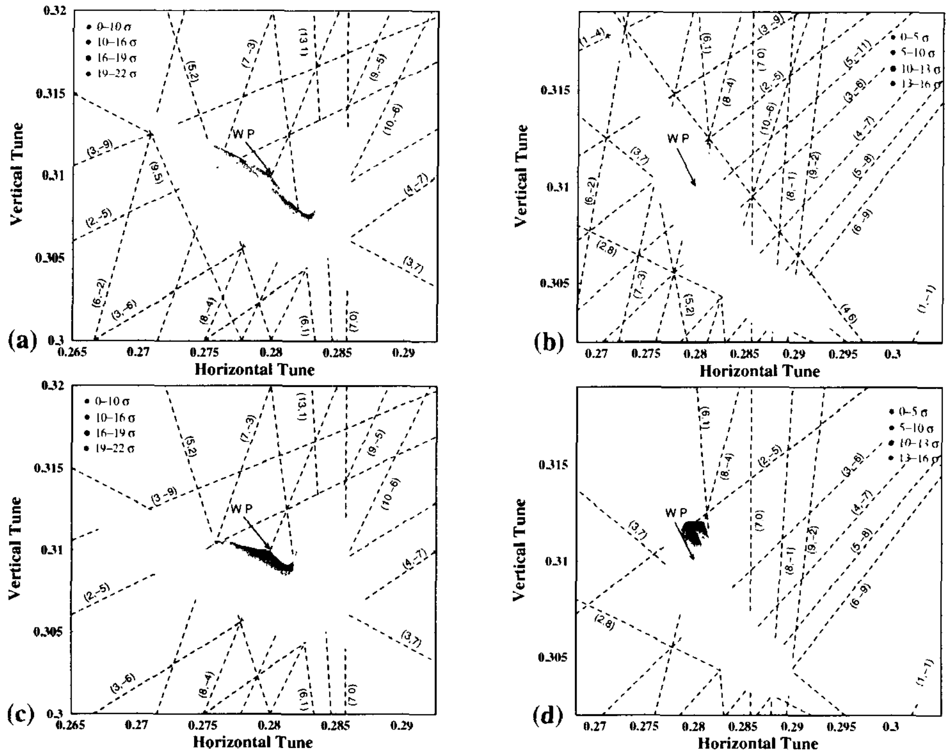

Figure 2: Frequency maps when the octupole and decapole components are not corrected, (top) and with the nominal correction scheme (bottom), for two different momentum spreads $\delta p/p = 0$ (left) and $\delta p/p = 7 \times 10^{-4}$ (right), and for the nominal working point $(Q_x, Qy) = (0.28, 0.31)$.

2 RESONANCE ANALYSIS

The machine considered was LHC optics version 6 with the integer tune split of four (63, 59). The LHC model constructed with MAD [8] includes systematic plus 1σ random errors in all the dipoles and can be considered as a worst case scenario. The normal sextupole error on the main dipoles has been corrected with spool pieces in every one of them around the machine. Short term tracking was performed with SIXTRACK [9] for two different working points: the nominal one $(Q_x, Q_y) = (0.28, 0.31)$ and another interesting candidate for the LHC operation $(Q_x, Q_y) = (0.21, 0.24)$ which has the same split between the tunes and the same distance to the tune space diagonal. This working point is closer to the 5th order resonance $(5, 0)$ on the horizontal plane and the 4th order $(4, 0)$ on the vertical plane. Apart from the 4D cases ($\delta p/p = 0$), it was also essential to perform tracking with a constant momentum deviation $\delta p/p = 7.5 \times 10^{-4}$, at approximately 75% of the full bucket size. As off-momentum particles cross areas where the dispersion is non-zero (typically 1-2m in the LHC), feed-down effects are generated by the multi-poles: at first order, the decapole will create an octupole, the octupole will create a sextupole, etc. The octupole produces a first order tune-shift, linear with the emittance. The decapole has a second order contribution (quadratic in the decapole strength and in the emittances). On the other hand, for off-momentum particles, the decapole gives an octupole feed-down and thus contributes to a first order tune-shift.

Frequency maps [5] were produced for all the correction cases. As an example, we present in Fig. 2, four maps issued by 4D (left) and 5D (right) tracking of 10000 particles for the nominal working point $(Q_x, Q_y) = (0.28, 0.31)$ and two different models: the non-corrected case (top) and the reference correction (bottom). The different colors in the maps represent different amplitude windows, up to 22 and 16 σ for the 4D and 5D case, respectively. As expected, the non-corrected case is very bad with respect to non-linear dynamics. The tune-shift is quite large especially for particles with large vertical amplitude (left corner of the plots). The most excited resonance seems the normal 7th order $(2, -5)$ and the 9th order $(3, -6)$. Their interplay with other 7th order and 10th order resonances, represented as crossings of lines in the map, can perturb severely the particle motion. For the 5D case, it is important to point out that their is a shift in the tune for "zero-amplitude" particles which is caused by the fact that the chromaticity is not zero (actually it is equal to -2). In that case, the particles are shifted up towards the normal 5th order $(1, -4)$ resonance, for the uncorrected case. Note also the trapped particles close the diagonal (resonance $(1, -1)$). The correction seems quite efficient, as the tune-shift is reduced, especially in the off-momentum case. For the other working point, maps were also constructed and dangerous resonances where identified. Especially the $(5, 0)$ resonance is approached by particles with high vertical amplitudes in

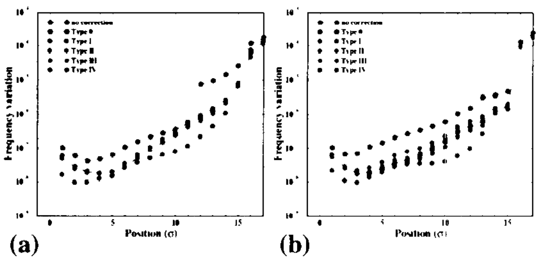

Figure 3: Evolution of the frequency variation averaged over all directions, with the particles' amplitude (in σ) for $\delta p/p = 7 \times 10^{-4}$, and for two different tunes (a) $(Q_x, Qy) = (0.28, 0.31)$ and (b) $(Q_x, Qy) = (0.21, 0.24)$.

the uncorrected case. The correction helps avoiding this resonance by reducing the detuning. For the other correction schemes the maps look quite similar. Especially the Type I scheme (every 2nd dipole) is quite good, even if the detuning seems to be bigger. In that case, and for motion close to the vertical plane, the tune-shift pushes the particles away from the dangerous crossing of resonances $((5,2)$ with $(2,-5)$ for the nominal working point and $(5,0)$ with $(1,-5)$ for the second one). In Fig. 3, we plot in logarithmic scale the frequency variation norm [5] averaged over the angles versus the amplitude, for both working points, for all correction schemes and for the non-zero momentum spread. It is confirmed by this plot that all the correction schemes are quite similar. The Type I (red dots) correction schemes seems to generate less perturbation for small amplitudes, in contrast with the Type II (every 2nd cell - pink dots), which seems slightly worse. In order to have some more insight about the resonance excitation with respect to the different correction schemes, we constructed 11th order 4D maps [6] for every correction case and post-processed the normal form results with GRR [7], in order to compute the resonance driving terms at a specific position of the phase space. In Fig. 4, we plot the resonance driving terms' norm (the sum of squares of all resonances up to 12th order) averaged over eleven directions in the phase space and computed at an amplitude of 12 σ. We should point out that 99.9 % of the contribution comes from resonances of order 7 and below and more then 85 % comes from the $(1,-1)$ resonance, whose phase averaged strength is also plotted in Fig. 4. There is a clear difference between the corrected and uncorrected cases, especially due to the contribution of all other resonances (left plots). On the other hand, the $(1,-1)$ resonance seems quite excited in all cases (right plots). This resonance was already identified to be one of the major dynamic aperture limitations the of LHC optics version 5, especially due to the integer tune-split of four (63,59). In particular, for the correction scheme II (every second cell), the strength of this resonance is even higher then in the uncorrected case. The change of the tune-split from four to five will most probably cure this undesirable effect. All the other correction schemes seem to have approximately the same resonance excitation, with the reference correction scheme being slightly better. A partial result from this representation is that the resonance

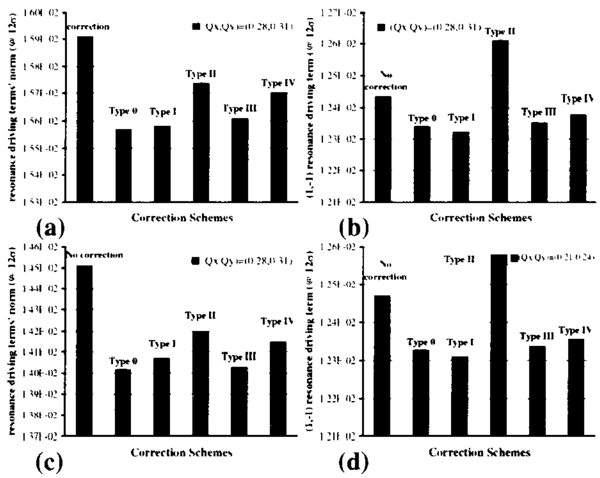

Figure 4: Resonance driving terms' norm (right) and driving term of the $(1,-1)$ resonance (left) extracted by 4D DaLie [6] maps, for two different working points $(Q_x, Qy) = (0.28, 0.31)$ and $(0.21, 0.24)$. The driving terms are evaluated at 12σ and averaged over eleven different directions of the phase space, with GRR [7].

excitation seems smaller for the working point (0.21,0.24) then for the nominal one. This can be also observed in the frequency variation (Fig. 3), where particles with the same amplitude have higher diffusion coefficient for the working point (0.28,0.31).

In conclusion, the different correction schemes seem equivalent from the point of view of non-linear dynamics, with the exception of Type II (every second cell) which is not as good as the others. It is thus confirmed that by using half of the correctors will be sufficient to correct the normal octupole and decapole errors in the LHC main dipoles, even if the reference case (correctors in every one of them) seems to be slightly better and, probably safer. Element by element long term tracking will be the ultimate test for confirming the above results.

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REFERENCES

[1] The LHC Study Group, CERN/AC/95-05 (LHC), 1995.

[2] Y. Papaphilippou, et al., LHC Project Report 253, 1998.

[3] F. Schmidt, G. Xu, LHC Project Note 98, 1997.

[4] J.-P. Koutchouk, private communication.

[5] J. Laskar, Physica D 67, 257, 1993; J. Laskar, D. Robin, Part. Accel. 54, 183, 1996; Y. Papaphilippou, PAC'99, New York, 1999.

[6] M. Berz, et al., Part. Accel., 24, 91 109, 1989; Berz, M., Part. Accel., 24, 109, 1989; E. Forest, The DaLie Code, unpublished, 1986; J. Irwin, et al., EPAC'94, London, 1994.

[7] Y. Papaphilippou, F. Schmidt, AIP C.P. 468, 1999.

[8] H. Grote, F.C. Iselin, CERN/SL/90-13, 1990.

[9] F. Schmidt, CERN/SL/94-56, 1994.